Numerical Analysis
Assignment One

Deadline October 11

Question 1. Perform the following computations (i) exactly, (ii) using three-digit chopping arithmetic, and (iii) using three-digit rounding arithmetic. (iv) Compute the relative errors in (ii) and (iii).

$$\textbf{a}.\quad \left(\frac{1}{3}-\frac{3}{11}\right)+\frac{3}{20};\qquad \textbf{b}.\quad \left(\frac{1}{3}+\frac{3}{11}\right)+\frac{3}{20}.$$

Question 2. Show that using k-digit rounding, a number will keep k significant digits.

Question 3. Count the numbers of operations for

$$f(x)=2x^4-x^3+4x^2-12x+1.2$$

and its nested form

$$f(x)=2*(((x-0.5)x+2)x-6)x+1.2.$$

Question 4. Find root(s) accurate to 10^{-5} for the function $f(x)=x^3+\cos(x)+1$.

a. Verify that there exists a zero of $f(x)$ on the interval $[-2,0]$.

b. How many iterations would it take when we apply the bisection method?

c. Use Newton's method with a initial approximation $p_0=-1$. Could $p_0=0$ be used? Why?

Question 5. Let $f(x)=10\cos(x)-x$. Verify that the interval $[0,3]$ contains a zero of $f(x)$. Then give a solution of $f(x)=0$ of six significant digit accuracy using (i) Newton's method with $p_0=1.5$, (ii) the Secant method with $p_0=1.5$ and $p_1=2$, respectively. Please also provide your Matlab codes.

Question 6. Consider a fixed-point problem $x=g(x)$. We have shown the existence of the fixed point provided that $|g'(x)|\le k<1$ for $a\le x\le b$ in the lecture. However, this is just a sufficient condition.

(i). Please give an example of the existence of fixed point of $g(x)$ that does not fulfil that condition.

(ii). Assume p is a fixed point of $g(x)$ and $|g'(p)|>1$. Show that the sequence $\{p_n\}_{n=0}^{\infty}$ generated by

$$p_n=g(p_{n-1})$$

doesnot converge to p for any given p_0.